Hierarchical Graph Traversal for Aggregate k Nearest Neighbors Search in Road Networks (Extended Abstract)*

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Abstract

A k nearest neighbors (kNN) query finds k closest points-of-interest (POIs) from an agent's location. In this paper, we study a natural extension of the kNN query for multiple agents, namely, the Aggregate k Nearest Neighbors (AkNN) query. An AkNN query retrieves k POIs with the smallest aggregate distances where the aggregate distance of a POI is obtained by aggregating its distances from the multiple agents (e.g., sum of its distances from each agent). We propose a novel data structure COLT (Compacted Object-Landmark Tree) which enables efficient hierarchical graph traversal and utilize it to efficiently answer AkNN queries. Our experiments on real-world and synthetic data sets show that our techniques outperform existing approaches by more than an order of magnitude in almost all settings.

1 Introduction

A kNN query [Zhong *et al.*, 2015] returns k points-of-interest (POIs) closest from an agent's location, e.g., find three closest gas stations to a taxi driver in a given road network. But in many applications (as e.g., ride-sharing [Stiglic *et al.*, 2015; Drews and Luxen, 2013]), it is important to obtain nearby POIs considering the locations of multiple agents. This motivates the consideration of aggregate k nearest neighbors (AkNN) queries [Yiu *et al.*, 2005; Zhu *et al.*, 2010], which are a natural extension of kNN queries that retrieve POIs considering their aggregate distance from all agents. For example, a group of friends might want to decide for a restaurant to meet such that the maximum distance any of them needs to travel is minimized. They can issue an AkNN query to find k restaurants with the smallest aggregate distances where the aggregate function is max.

More formally, given a POI p and a set of agents Q, the aggregate distance of p from the agents is $d_{agg}(Q,p) = agg(d(q_i,p), \forall_{q_i \in Q})$ where $d(q_i,p)$ denotes the road network distance (e.g., travel time, path length) from an agent q_i to p and $agg()$ is an aggregate function. For example, when the aggregate function is sum, $d_{agg}(Q,p) = \sum_{q_i \in Q} d(q_i,p)$ and when the aggregate function is max, $d_{agg}(Q,p) = \max_{q_i \in Q} d(q_i,p)$. An A$k$NN query returns k POIs with the smallest aggregate distances.

Yiu *et al.* [Yiu *et al.*, 2005] solve the AkNN query using a hierarchical search on the road network. They utilize an R-tree [Guttman, 1984] which recursively divides POIs into subsets by Minimum Bounding Rectangles (MBRs). During search, a lower-bound aggregate distance for all POIs in a child R-tree node is computed using the Euclidean distances to its MBR. The algorithm visits the most promising R-tree branches to pinpoint result POIs. However, Euclidean distance is only a loose lower-bound especially on metrics like travel time, making the heuristic less efficient. Moreover, the inefficiency is exacerbated for AkNNs as the error will also be aggregated. Landmark Lower-Bounds (LLBs) are better heuristics than Euclidean distance [Goldberg and Harrelson, 2005], however, there does not exist a hierarchical data structure to compute minimum LLBs to groups of POIs in the same way as R-trees using Euclidean distance.

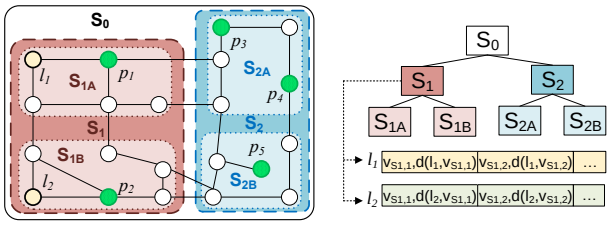
In this paper, we address the above mentioned issues and present two hierarchical indexes, SL-Tree and COLT, which add significantly more landmarks than previous methods resulting in tighter bounds, while keeping the indexes reasonably small. COLT is the first index to support hierarchical traversal of the road network using landmark lower-bounds. We propose a heuristic search algorithm to answer AkNN queries for convexity-preserving aggregate functions such as sum and max. Our experiments demonstrate that our techniques achieve up to three orders of magnitude improvement.

2 Proposed Indexes: SL-Tree and COLT

First, we give some background. A road network is a graph $G = (V, E)$ where V is a vertex set and E is an edge set. Each edge $(u, v) \in E$ connects two vertices with weight $w(u, v)$ representing any real positive metric, e.g., length, travel time, toll cost of the edge etc. Network distance $d(s, t)$ is the minimum sum of weights connecting vertices s and t. We consider queries and POIs (also called objects in this paper) to be on graph vertices for simpler exposition.

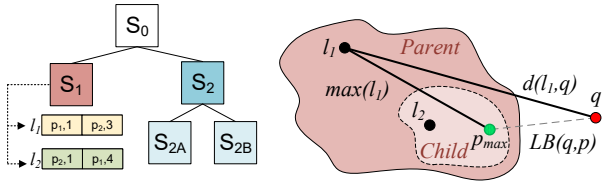
Landmark lower-bounds, also called differential heuristics [Goldenberg *et al.*, 2011], involve selecting a set L of m "landmark" vertices and then pre-computing distances from

*This is an abridged version of a paper that won the best paper award at ICAPS 2020 [Abeywickrama *et al.*, 2020].



(a) SL-Tree Subgraph Partitions (b) SL-Tree Node Hierarchy

Figure 1: Subgraph Landmark Tree (SL-Tree)



(a) COLT Hierarchy (b) COLT Lower-Bounds

Figure 2: Compacted Object-Landmark Tree (COLT)

each landmark to all vertices in V. (1) gives a lower-bound on network distance $dist(q,p)$ using the triangle inequality. The maximum lower-bound over all m landmarks given by (2) gives a tighter lower-bound and is typically accurate even for small m [Goldberg and Harrelson, 2005].

$$LB_{l_i}(q,p) = |d(l_i,q) - d(l_i,p)| \leq d(q,p) \quad (1)$$

$$LB_{max}(q,p) = \max_{l_i \in L}(|d(l_i,q) - d(l_i,p)|) \quad (2)$$

For monotonic aggregate functions, the aggregate of lower-bound distances to object p from each query vertex in Q is a lower-bound $LB_{agg}(Q,p)$ on aggregate distance $d_{agg}(Q,p)$ [Yiu *et al.*, 2005], i.e., $LB_{agg}(Q,p) = agg(LB(q_1,p),\ldots,LB(q_{|Q|},p)) \leq d_{agg}(Q,p)$.

Road network index. We first introduce the Subgraph-Landmark Tree (SL-Tree) to index the road network. The SL-Tree is a supporting index that we use to construct our object index efficiently. Each node in the SL-Tree represents a subgraph of the road network with the root being G. G is recursively partitioned into b disjoint subgraphs of equal size, stopping when a subgraph has no more than α vertices. Figure 1a shows such a partitioning for $b = 2$ and $\alpha = 6$ with the corresponding SL-Tree shown in Figure 1b. Note that we refer to tree *nodes* and road network *vertices*.

For each node n_T, we select m of its vertices as *local landmarks*, e.g., l_1 and l_2 for $m = 2$ for S_1 in Figure 1a (landmarks for other nodes are omitted for clarity). We then compute distances from each l_i to every vertex in n_T's subgraph using Dijkstra's search, which are then stored in *distance list* DL_i. Figure 1b shows the distance lists for the landmarks of S_1 (those for other nodes are again omitted). Note that subgraphs are stored implicitly by mapping each road network to the SL-Tree leaf node that contains the vertex.

Object index. The SL-Tree can then be used to efficiently construct our object index, the Compacted Object-Landmark Tree (COLT). COLT is a carefully compacted version of the SL-Tree for object set P. Compaction ensures that there are m local landmarks for at most λ objects, to increase the likelihood of finding a tighter lower-bound for more objects. Note that the SL-Tree is shared between construction of all COLT indexes, i.e., for many different object sets.

Given SL-Tree T, COLT index C is constructed by visiting nodes in T in a top-down manner and creating corresponding nodes in C. Let n_T be the currently visited node in T (initially the root). A corresponding node n_C is created in C for n_T. Let λ be the maximum number of objects in a leaf node for COLT. If n_T contains more than λ objects, the search expands to its children. Otherwise, the search is pruned at n_C, which becomes a leaf-node of COLT. For the new leaf n_C, an *Object Distance List* ODL_i is created in n_C for each landmark l_i in n_T. These are simply the distance lists of n_T, except with only the distances for object vertices from P. Any interior nodes with only one child are merged with the child (keeping the child's more localized landmarks). Figure 2a shows a COLT index for $\lambda = 2$ constructed from the SL-Tree in Figure 1b based on the 5 object vertices (green shaded vertices) in Figure 1a. Note that S_{1A} and S_{1B} were removed as construction was pruned at S_1 due to its number of objects (ODLs of other nodes are omitted for clarity). While we have chosen $\lambda < \alpha$ to simplify the example, generally $\lambda \geq \alpha$ is required to guarantee no leaf node and therefore no ODL contains more than λ objects. For example, if every vertex in a leaf node is an object there will be α objects in its ODL.

Each object distance list ODL_i of leaf node n_{leaf} in C is sorted on distance. In non-leaf nodes n_C, we only store the minimum distance min_{n_C,l_i} and maximum distance max_{n_C,l_i} to any object in the node from each of its landmarks l_i. These are computed using distance lists in corresponding SL-Tree nodes. Next, we use this information to compute lower-bounds to nodes and traverse the hierarchy.

2.1 Lower-Bound Heuristic for Graph Traversal

Similar to the lower bound in (1), we can compute a lower bound for all objects contained within a node n_C in COLT index C by (3) for one landmark $l_i \in n_C$. (4) gives the best lower-bound over all m landmarks of n_C:

$$LB_{l_i}(n_C,q) = \begin{cases} d(l_i,q) - M^+ & \text{if } d(l_i,q) \geq M^+ \\ M^- - d(l_i,q) & \text{if } d(l_i,q) \leq M^- \\ 0 & \text{else} \end{cases} \quad (3)$$

$$LB_{max}(n_C,q) = \max_{l_i \in n_C}(LB_{l_i}(n_C,q)) \quad (4)$$

Thereby, $M^- := min_{n_C,l_i}$ and $M^+ := max_{n_C,l_i}$ for n_C are already available in COLT. However, for non-root nodes, $d(l_i,q)$ in (3) is only available if l_i and q are in the same subgraph. Pre-computing this distance for all V and landmarks is infeasible given the space implications. Alternatively, computing $d(l_i,q)$ on the fly using another technique is expensive and may be wasteful if the node does not contain results. Interestingly, (3) still holds if we replace the distances with lower-bound $LB(l_i,q)$ and upper bound $UB(l_i,q)$, as in

(5). The distance lists of the root or lowest common ancestor SL-Tree node can be used to compute the best $LB(l_i, q)$ and $UB(l_i, q)$ by (2) and its upper-bound equivalent (by adding rather than subtracting distances), respectively. Choosing the tightest over all landmarks of n_C gives an inexpensive and accurate bound even with a few landmarks:

$$LB_{l_i}(n_C, q) = \begin{cases} LB(l_i, q) - M^+ & \text{if } LB(l_i, q) \geq M^+ \\ M^- - UB(l_i, q) & \text{if } UB(l_i, q) \leq M^- \\ 0 & \text{else} \end{cases} \tag{5}$$

Landmark choices. We first partition the plane into m equally sized slices around the Euclidean center of the subgraph and then choose one border vertex as a landmark from each slice. If a slice has no borders, we choose the slice vertex furthest from the Euclidean center.

Effective lower-bounds. The accuracy of COLT's inexpensive lower-bounds increases as we delve deeper into the hierarchy. In Figure 2b, let us say we use l_1 and its maximum object distance to compute a lower-bound for the child node. At the lower level, we may use the child's landmarks like l_2, which are local to the objects and more likely to produce a better lower-bound. This lets us differentiate tree branches and pinpoint the most promising candidates. Next, we utilize this as a decoupled heuristic for AkNN querying.

Complexity. COLT takes $O(|P|)$ space (linear to the input) and $O(|P| \log |P|)$ time, while SL-Tree space and time only increase by a factor of $O(\log |V|)$ over ALT. We refer the reader to [Abeywickrama *et al.*, 2020] for full derivation.

3 Query Algorithm for A*k*NN Search

We utilize COLT to retrieve candidate objects (POIs) likely to be AkNN results, by their lower-bounds using a novel property of Object Distance Lists (ODLs) as explained next.

3.1 Object Distance Lists and Convexity

Note that (1) can be expressed as an absolute value function of form $f(x) = |C - x|$ for some landmark l. Here, C is the constant distance $d(l, q)$ between the landmark l and the query point q, and x is a variable distance $d(l, p)$ depending on the object $p \in P$. Since absolute-value functions are convex, $f(x)$ is minimized for x closest to C. This property is useful to find the minimum lower-bound in an Object Distance List ODL for the landmark l for a single query vertex q. Since ODL essentially stores the domain of x for all objects in the node, the minimum lower-bound for the landmark l can be found by searching ODL for $d(l, c)$ closest to $d(l, q)$ for some object $c \in ODL$. Since ODL is sorted, this is possible using a modified binary search, as observed by [Abeywickrama and Cheema, 2017], in only $O(\log \lambda)$ time.

Finding the minimum lower-bound *aggregate* distance is complicated by the presence of multiple query locations $q_i \in Q$. But we know that the aggregate of *lower-bound* distances from each query vertex q_i is a lower-bound on aggregate distance for monotonic functions [Yiu *et al.*, 2005]. Therefore, the function to minimize becomes $f(x) = agg(|C_1 - x|, \cdots, |C_n - x|)$ for a monotonic aggregate function agg where C_i is $d(l, q_i)$ for the given landmark l and a query $q_i \in Q$. At first glance, this might suggest we need to search ODL for multiple values (i.e., once for each C_i) to find the object with minimum aggregate lower-bound. Surprisingly, it is not necessary for aggregate functions that preserve convexity. Moreover, we find that the most widely used functions, max and sum, do preserve convexity.

Specifically, once the minimum x^* of the function $f(x)$ is found, iteratively retrieving the object that gives the next smallest lower-bound simply requires checking the element to the right or left of x^* in ODL, due to the convexity of the function. However, unlike the single query case, finding the minimum of $f(x)$ is not obvious for aggregate kNN queries. Below, we show how to find the minimum for two common aggregate functions, max and sum.

Lemma 1. *Consider the aggregate function defined by the sum of a set of absolute functions $f(x) = sum(|C_1 - x|, \cdots, |C_n - x|)$. The minimum x^* of $f(x)$ is the median value of the constants $C_1, ..., C_n$.*

Lemma 2. *Consider the aggregate function defined by the maximum of a set of absolute functions $f(x) = max(|C_1 - x|, \cdots, |C_n - x|)$. The minimum x^* of $f(x)$ is $\frac{C_{min} + C_{max}}{2}$, i.e., the average of the minimum and maximum constants.*

We omit the proofs of Lemmas 1 and 2 which can be found in the full version of this paper [Abeywickrama *et al.*, 2020].

3.2 Algorithm

We use hierarchical graph traversal on the COLT index to guide us towards ODLs most likely to contain AkNN results. Our algorithm maintains a priority queue $\mathcal{PQ}$ containing objects and COLT nodes keyed by their aggregate lower-bound distances from Q. The algorithm iteratively extracts the minimum lower-bound queue element and terminates when either $\mathcal{PQ}$ becomes empty or when the key of the extracted element is at least equal to D_k where D_k is the aggregate distance of the k-th AkNN found by the algorithm so far.

If an object is extracted from $\mathcal{PQ}$, its exact aggregate distance is computed and the result set R and D_k are updated accordingly. If a non-leaf node is extracted then each of its child c is inserted in $\mathcal{PQ}$ with key set to c's aggregate lower-bound score computed according to (4) and (5). If a leaf node n is extracted from $\mathcal{PQ}$, the algorithm chooses an object p with the smallest aggregate lower-bound among the previously unseen objects in n and inserts p in $\mathcal{PQ}$. The node n is re-inserted in $\mathcal{PQ}$ if the algorithm has not already seen all objects in it. The object p in the node n is chosen as follows. If n is seen for the first time by the algorithm, p is chosen using a binary search to find the minimizing list index given by Lemma 1 or 2. Otherwise, p must be an object on the right or on the left of the previously seen objects in this leaf node (due to the convexity of the function), thus, is retrieved in $O(1)$.

4 Experiments

We use the continental US road network that contains $23,947,347$ vertices and $57,708,624$ travel time edges combined with 8 real POI sets (see Table 1) for the US from OpenStreetMap provided by [Abeywickrama *et al.*, 2016]. For

Parameter	Values
k	1, 5, **10**, 25, 50
d	1, 0.1, 0.01, **0.001**, 0.0001
A (%)	1, 5, **15**, 50, 100
$\|Q\|$	2, 4, **8**, 16, 32
Real-World POI Set ($\|P\|$)	Schools (160,525), Parks (69,338), Fast Food (25,069), Post Office (21,319), Hospitals (11,417), Hotels (8,742), Universities (3,954), Courthouses (2,161)

Table 1: Parameters (defaults in bold if applicable)

sensitivity analysis, we generate synthetic POI sets chosen uniformly at random for density d where d=$|P|/|V|$. We use A to denote a connected subgraph of G with $A\%$ of the total vertices $|V|$. Query vertices are then chosen uniformly at random from the $A\%$ subgraph which represents how "local" a group of query locations are [Yiu *et al.*, 2005]. We show the results over 500 queries considering max function. The results for sum can be found in [Abeywickrama *et al.*, 2020].

We compare our algorithm against the Incremental Euclidean Restriction (IER) [Yiu *et al.*, 2005] and a concurrent expansion approach which adapts the state-of-the-art kNN heuristic based on Network Voronoi Diagrams (NVDs) [Abeywickrama and Cheema, 2017] for AkNN queries. To ensure fairness, each technique employs Pruned Highway Labeling (PHL) [Akiba *et al.*, 2014], one of the fastest network distance computation techniques, to compute network distances in each approach – hence techniques are named IER-PHL, NVD-PHL, and COLT-PHL. SL-Trees and COLT use branch factor $b = 4$, maximum object distance list size $\lambda = 128$ (which is also α) and $m = 4$ landmarks per node for ideal performance vs. index size. NVD uses the ALT index [Goldberg and Harrelson, 2005] with $m = 16$ random landmarks to compute LLBs, which we also use as it is essentially the root of an SL-Tree. All experiments were conducted using memory-resident indexes for fast query processing.

Query performance on real-world data. Figure 3 depicts query time on real-world POI datasets, with the number of objects increasing from left to right. COLT significantly outperforms the other methods across the board, with up to two orders of magnitude improvement. COLT tends to improve more on larger POI sets, where it is more difficult to distinguish between objects.

Query performance on synthetic data. A sensitivity analysis of the query times on the synthetic benchmark sets with respect to the object density d, the value of k, the number of

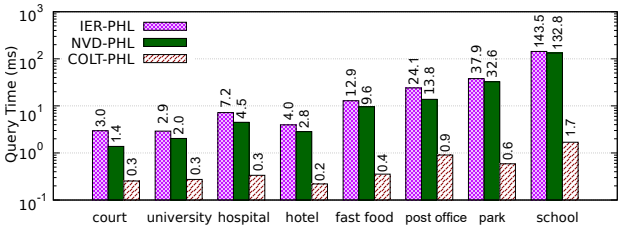
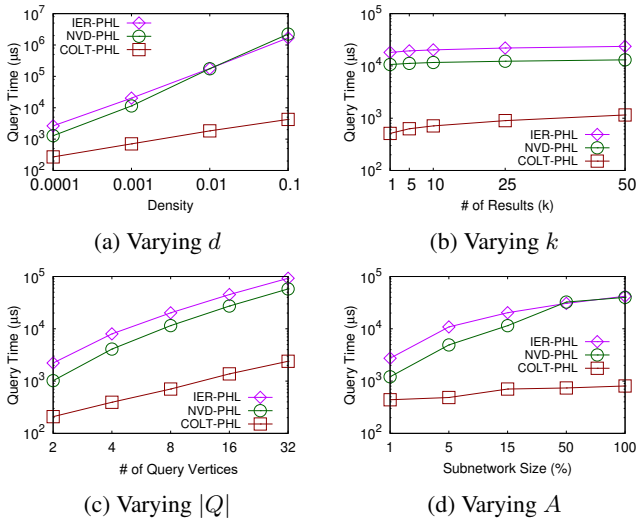


Figure 3: Performance on different real-world POI sets



(a) Varying d (b) Varying k

(c) Varying $|Q|$ (d) Varying A

Figure 4: Performance on synthetic data

	Network Indexes			Obj. Indexes (d=0.001)		
	ALT	SL-T	PHL	COLT	NVD	R-T
Time	71s	25m	25m	63ms	11s	6ms
Space	1.4GB	4.6GB	16GB	0.9MB	28MB	0.9MB

Table 2: Preprocessing cost (SL-T is SL-Tree & R-T is R-Tree)

query vertices $|Q|$, and the percentage A of graph vertices in a subgraph from which we choose query vertices is provided in Figure 4. This reaffirms the observations on real-world data and shows the improvement of COLT is consistent across varying parameters, reaching up to 3 orders magnitude.

Preprocessing cost. Table 2 shows preprocessing cost for the road network and object indexes. The SL-Tree consumes greater space than ALT, but not significantly so. Moreover, only the root node of the SL-Tree is required for query processing, which has the same index size as ALT. The indexing time is comparable to PHL, which possesses one of the fastest pre-processing times for high-performance indexes [Akiba *et al.*, 2014]. COLT is significantly smaller and faster to construct than NVDs and is comparable to R-trees as both have space complexity linear to the input.

5 Conclusion

We present two novel light-weight indexes, SL-Tree and COLT, and design an efficient hierarchical traversal algorithm to solve AkNN queries for any convexity preserving aggregate function. Our experiments on real-world and synthetic datasets show that the proposed approach outperforms existing techniques by up to three orders of magnitude.

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